

Lecture 11

Cook-Levin Theorem (contd.), Search vs Decision

Constructing the ϕ_x

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$$x \in L \iff \exists u \in \{0,1\}^{p(|x|)}, \text{ s.t. } M(x, u) = 1$$

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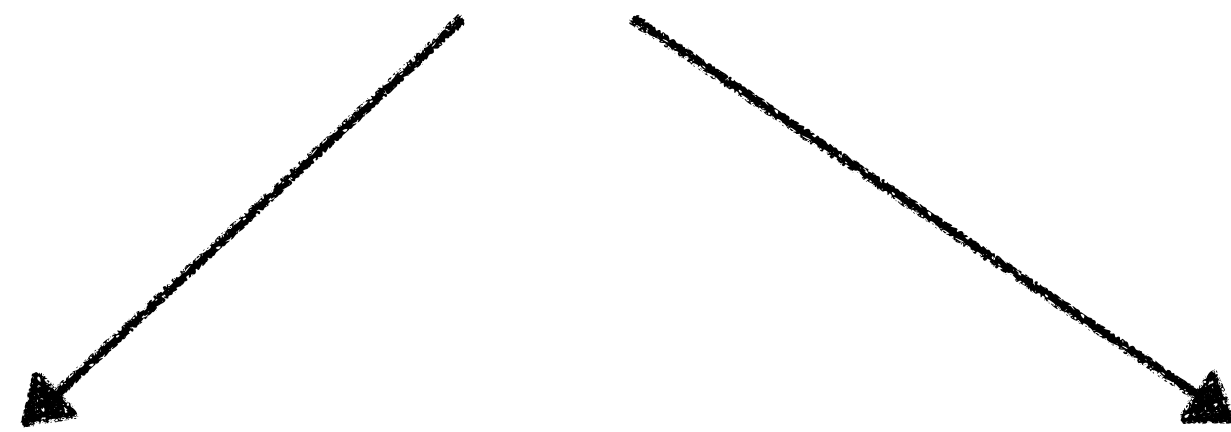
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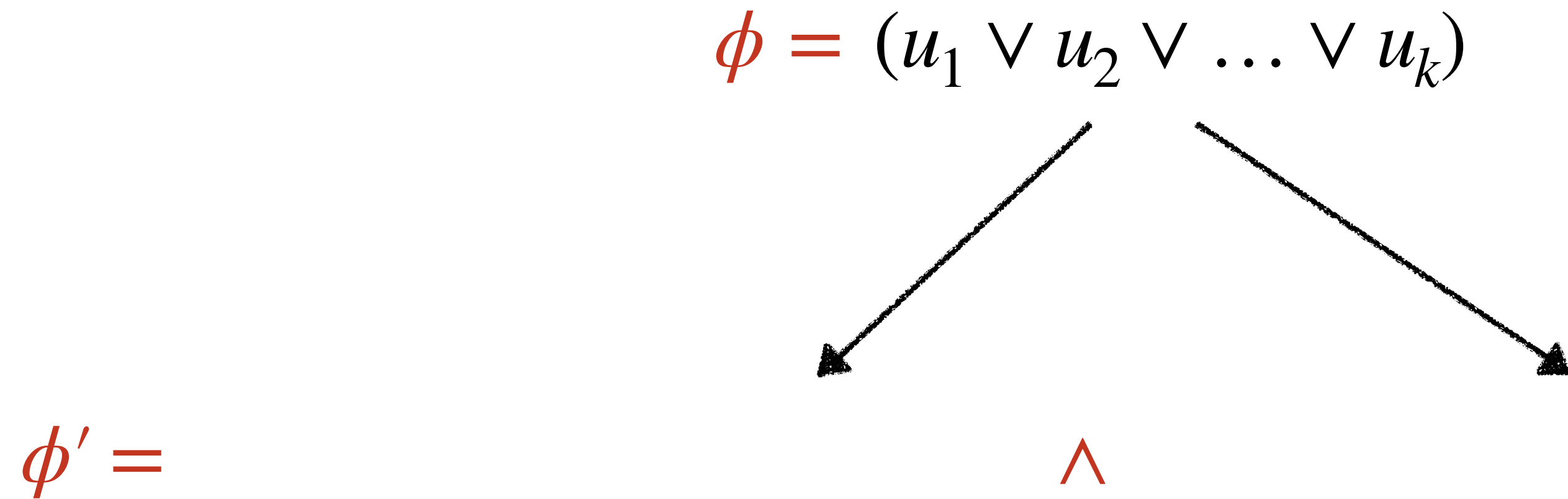
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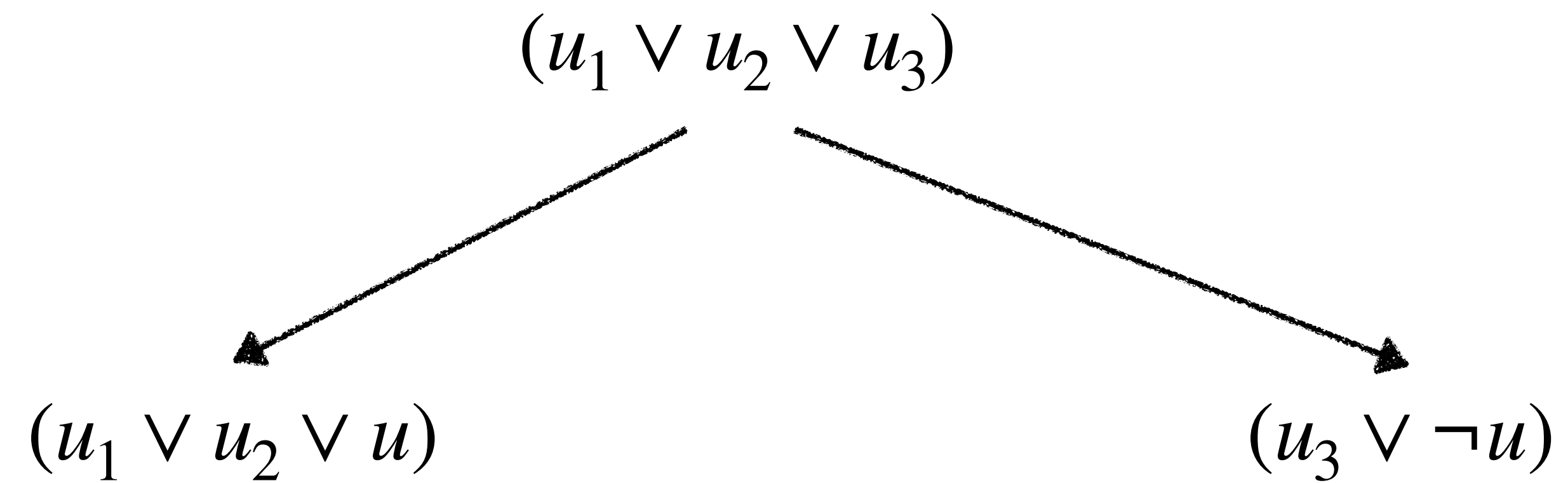
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Isn't 2SAT also NP-Complete?

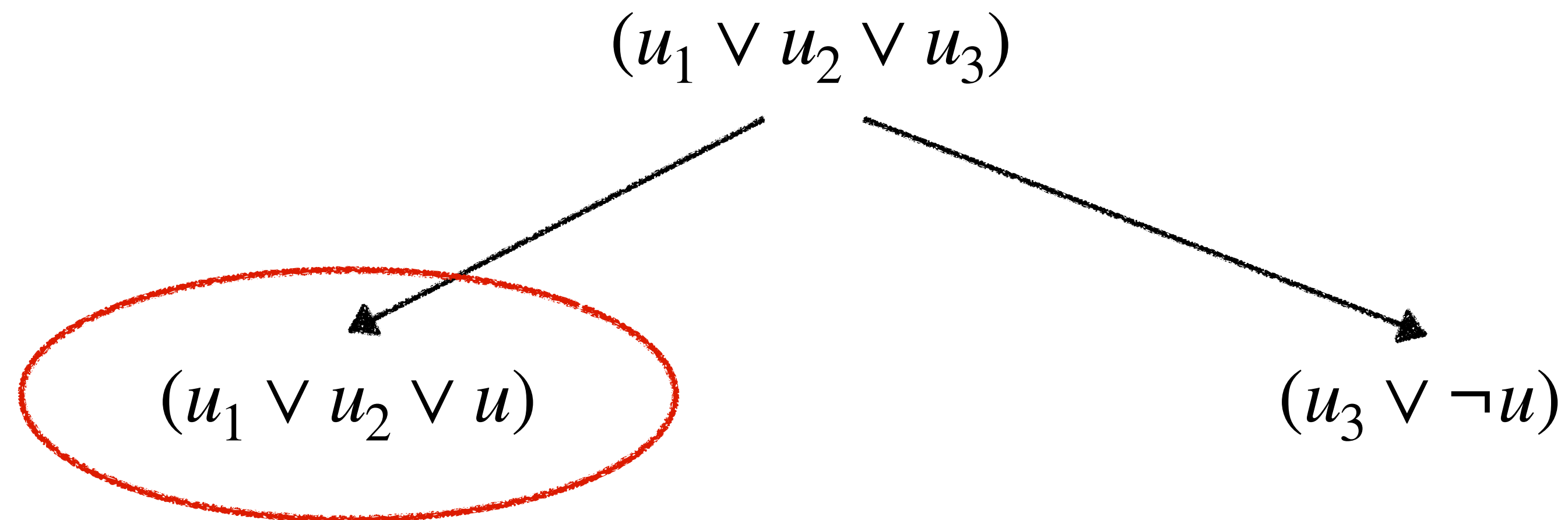
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Create a polytime TM B that on input ϕ , finds a satisfying assignment for ϕ if it exists.

Idea: Let $\phi = (u_1 \vee u_2) \wedge (\neg u_2 \vee \neg u_3) \wedge (\neg u_1 \vee u_3)$

$$\phi_{u_1=0} = (0 \vee u_2) \wedge (\neg u_2 \vee \neg u_3) \wedge (1 \vee u_3) = (u_2) \wedge (\neg u_2 \vee \neg u_3)$$

$$\phi_{u_1=1} = (1 \vee u_2) \wedge (\neg u_2 \vee \neg u_3) \wedge (0 \vee u_3) = (\neg u_2 \vee \neg u_3) \wedge (u_2)$$

If ϕ is satisfiable then either $\phi_{u_1=0}$ or $\phi_{u_1=1}$ is satisfiable.

Search vs Decision

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- 1) First $|x|$ bits of $y = x$. (Linear size ϕ_1 . If $x = 101$, then $\phi_1 = (Y_1) \wedge (\neg Y_2) \wedge (Y_3)$)
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